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Collapse of a Spherical Cloud:

We now consider the spherical collapse scenario in greater detail. Lets consider a region containing a mass of $M = M_\odot$ that becomes unstable to gravitational collapse at some stage in the collapse of a larger cloud. Given the mass and a typical initial temperature $T_i \sim 50\,K$, we can estimate the initial radius for a virialized clump:

$$\frac{1}{2} M_\odot v_{rms,i}^2 \sim \frac{3}{5} \frac{G M_\odot}{R_i^2}, \quad v_{rms,i} \sim 10^5\ cm\,s^{-1} \Rightarrow R_i \sim 2\times10^5 R_\odot$$

$$L_i = 4\pi R_i^2 \sigma T_i^4 \sim 10^2 L_\odot$$

The initial composition is mostly Hydrogen (in molecular form, H_2) and Helium with smaller quantities of heavier elements.

The cloud will contract isothermally initially. The temperature remains constant, while R decreases, which implies a decrease in the luminosity L.

Note that the pressure gradient per unit mass is $\sim \frac{P}{\rho R}$ $\sim \frac{T}{R}$, while the gravitational force per unit mass is $\frac{GM}{R^2}$.
As a result, gravity dominates more and more during an isothermal contraction. Ignoring pressure, the evolution of the radial distance r of a shell follows:

$$\ddot{r} = -\frac{GM_r}{r^2}$$

The solution to this equation is:

$$r = r_0 \cos^2 \zeta \quad , \quad t = \left(\frac{8\pi G \rho_0}{3}\right)^{-\frac{1}{2}} \left(\zeta + \frac{1}{2}\sin 2\zeta\right)$$

Here r_0 is r at $t=0$ (which corresponds to $\zeta = 0$), and ρ_0 is the initial density (uniform distribution is assumed).
It is seen that at $\zeta = \frac{\pi}{2}$ we have $r=0$ regardless of r_0. This gives the free-fall time:

$$\boxed{t_{ff} = \left(\frac{3\pi}{32 G \rho_0}\right)^{\frac{1}{2}} \simeq \left(\frac{\rho_0}{2\times10^{-16}\,g\,cm^{-3}}\right)^{-\frac{1}{2}} 4.7\times10^{3}\ yr} \quad *$$

The whole sphere contracts to a point in a finite time t_{ff}.

For a dense molecular cloud with $T \sim 50\,K$ and $M = M_\odot$, we find $t_{ff} \simeq 4700\,yr$.

The density of the cloud will not be constant in any realistic case. It will be larger near the center, which implies t_{ff} will be smaller for the inner core. In consequence, a central concentration will be formed during the collapse that enhances itself. The central region will become opaque first and will make a transition to adiabatic contraction. The temperature increases during this stage, which eventually halts contraction of the central region. This results in the formation of hydrostatic core (usually called a protostar), which is surrounded by freely falling gas. The problem then reduces to that of accretion onto a central object.

The density profile of the accreting outer regions depends on whether the gravitational force is dominated by the central concentration or by the self-gravity of the cloud. At the beginning, the gravitational force of the central object is negligible. The density profile is then given by $\rho(r) \propto r^{-2}$. The mass of the core grows in time and it will eventually dominate the gravitational force. At that point the density profile of the clouds changes over to $\rho(r) \propto r^{-3/2}$.

When the infalling material hits the hydrostatic core, a shock wave will develop because the speed of the material exceeds the local sound speed. An estimate of the accretion rate is obtained as:

$$\dot{M} \sim \frac{M_r}{t_{ff}(r)}$$

This considers the rate of increase in the core mass as a result of the infall of the mass contained within the radial distance r.

After using the expression in equation * for t_{ff}, and definition of the escape velocity at r:

$$v_{esc}(r) \approx \frac{GM_r}{r}$$

We find:

$$\dot{M} \sim \frac{v_{esc}^3(r)}{G}$$

The supersonic accretion occurs if $v_{esc}(r)$ exceeds the local sound speed. For a supersonic accretion we have:

$$\dot{M} \sim \frac{c_s^3}{G} \approx 2 \times 10^6 \left(\frac{T}{10K}\right)^{3/2} M_\odot\ yr^{-1}$$

The density profile of the cloud in this case is given by:

$$\rho(r) \sim \frac{c_s^2}{2\pi G r^2}$$

Various compositional changes occur in the core. When the central temperature is $T_c \sim 2000$ K, the Hydrogen molecules are dissociated. During this phase the supplied energy goes into dissociation instead of increasing the kinetic energy. Then, since the mass of the core increases due to accretion, the core becomes unstable again and the protostar will collapse again. Once the molecular

dissociation is complete, the core temperature will begin to increase again. Note that $\gamma_{ad} = \frac{7}{5}$ before dissociation (diatomic gas), while $\gamma_{ad} = \frac{5}{3}$ after the completion of dissociation (monatomic gas).

A sequence of the events at the core can be given as follows (Consider a $1 M_\odot$ cloud):

– The central region becomes opaque when $\rho_c \sim 10^{-13}$ g cm^{-3} and $T_c \sim 50$ K. The radius of the cloud at this time is ~ 5 AU.

- The central core reaches hydrostatic equilibrium when $\rho_c \simeq 2\times10^{-10}$ g cm^{-3} and $T_c \simeq 170$ K.

- The central core becomes unstable at a temperature $T_c \sim 2000$ K, because of H_2 dissociation. It will collapse again (isothermally) until the molecular dissociation is complete.

- There will be further unstabilities at higher temperatures when ionization of H and He happens.

One can estimate the radius of cloud once the ionization process is complete. The total energy needed is;

$$E_{ion} = (N_A X) E_H + \frac{1}{4} (N_A Y) E_{He} + \frac{1}{2} (N_A X) E_D$$

$E_H = 13.6$ eV , $E_{He} = 78.98$ eV , $E_D = 4.48$ eV

N_A: Avogadro's number

Here E_{He} is the sum of energies for $He \to He^+$ and $He^+ \to He^{++}$,

and E_D is the dissociation energy of H_2.

Setting E_{ion} equal to half of the gravitational energy released in the collapse from initial radius to R (valid for a quasi-static evolution), we find:

$$\boxed{\frac{R}{R_\odot} \sim \frac{43.2 \frac{M}{M_\odot}}{1-0.2X} \sim 50 \frac{M}{M_\odot} = 50}$$

The corresponding central temperature is given by:

$$\boxed{T_c \sim 3 \times 10^5 \sim (1-0.2X) \sim 10^5 \text{ K}}$$

Then the change in radius (for $M = M_\odot$) is from $\sim 10^5 R_\odot$ to $\sim 50 R_\odot$, with a corresponding change of $\sim 10^2 L_\odot$ to $\sim 10^{-4} L_\odot$ in the luminosity (the free-falling cloud is still isothermal).